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Espresso

A sip of filtered research

CAMBRIDGE Mathematics

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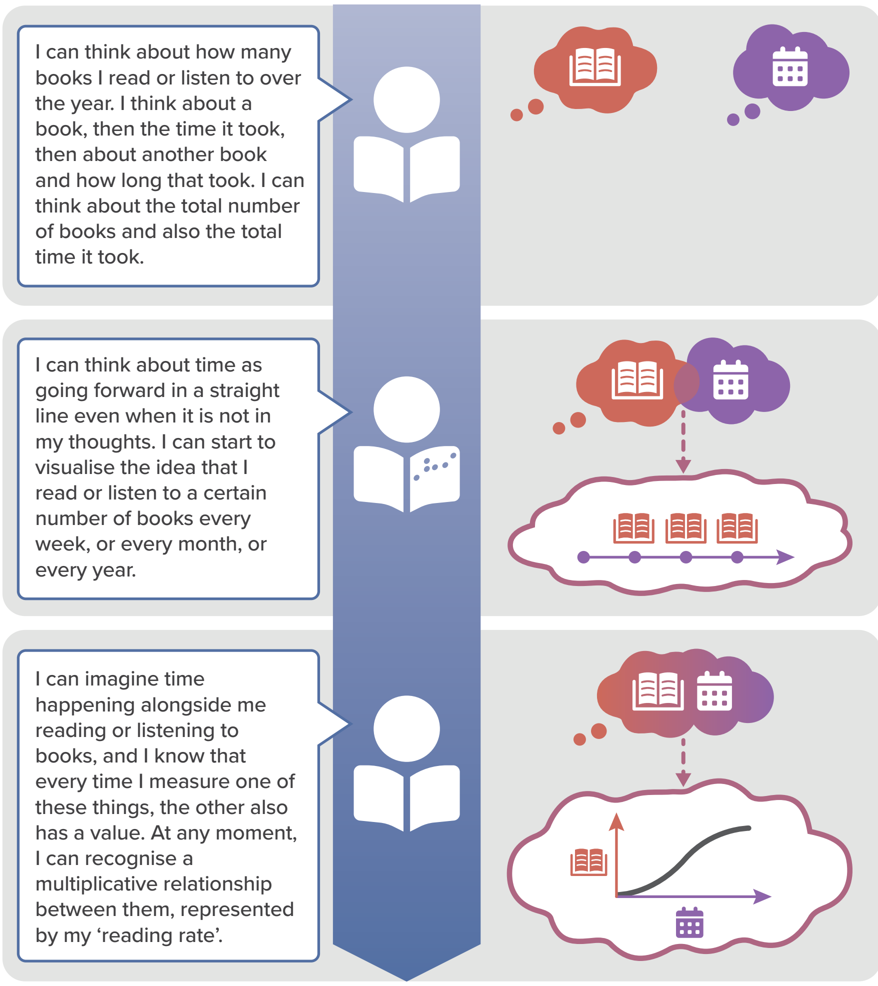
Talking point

What does research suggest about covariational reasoning?

In summary

- Covariational reasoning can be explored across the breadth of mathematical activity from early primary onwards
- Students can be given opportunities to develop covariational reasoning through exploring linear functions and rates; making and critiquing statements; and considering association, correlation and scatter plots
- Focusing on plotting or comparing pairs of values may limit opportunities to develop covariational reasoning
- Covariational reasoning in statistical contexts requires students to also coordinate the idea of signal and noise, and this can be challenging
- It may be beneficial for students developing covariational reasoning to use either time or a counting sequence of numbers as the second variable
- Exploring covariation through multiple representations is important; students may find it easier to use graphs at first, especially non-standard ones
- Useful activities might include highlighting fallacies and misconceptions, using modelling, and using real data and/or familiar contexts
- Software tools can be used to support students' exploration of covariation by allowing more time to focus on exploring patterns

Development of students' covariational reasoning



Adapted from ideas in Thompson & Carlson (2017)[1]

1 Covariational reasoning is playing with two quantities that change and exploring how they relate to one another as they change.[2] It involves *coordinating* (tracking how changes in one thing might impact another related thing, and vice versa) two variables (quantities) as their values change simultaneously,[1] which is relevant to students exploring ideas in number, algebra, geometry and statistics, and making connections between them. In statistics, covariational reasoning relates to exploring bivariate (paired) data. 'Covariational reasoning activities may involve exploring linear functions and rates; considering association, correlation and scatter plots; and making and critiquing statements of the form 'doing more/less of x results in more/less of y.[3] As a result, covariational reasoning is important across a range of disciplines, including psychology and other sciences.

Implications:

Covariational reasoning can be applied across the breadth of mathematical activity, allowing students to make rich connections

Students can be given opportunities to develop covariational reasoning through exploring linear functions and rates; considering association, correlation and scatter plots; and making and critiquing statements of the form 'doing more/less of x results in more/less of y'

2 Children as young as six can begin to develop a *covariational view* by exploring relationships between quantities as they change, but are often provided with limited opportunities to do so, focusing instead on patterns in a single variable.[4] Overemphasising a static approach, often a result of drawing graphs without technological support, may mean that students conceptualise a graph or function as a series of isolated points only, rather than recognising that patterns of covariation continue in the gaps.[5,6] In statistics-related contexts, this can be even harder, as students have to think about both the covariation based in the underlying relationship (the model without the randomness), and the statistical variation present in the data (what happens in real life when the randomness is present). This leads to a 'signal and noise' model of covariation in statistical contexts; for instance, when using lines of best fit.[7]

Implications:

Students can be given opportunities to explore covarying quantities from early primary education onwards

Focusing on plotting or comparing pairs of values may limit opportunities to develop covariational reasoning

Covariational reasoning in statistical contexts requires students to also coordinate the idea of signal and noise, and this can be challenging

3 Early experiences of covariation can be developed alongside exploration of pattern by coordinating a pattern with the counting sequence (perhaps using number cards; see Espresso 47 for more detail).[8] Thinking about covariation by understanding variation as happening over time and explicitly thinking about the relationship between the two may support the development of covariational reasoning.[9,10] Students are usually more successful at describing or identifying an association between two variables in graphs or charts rather than in tables.[11] Activities which may support developing concepts related to covariational reasoning tend to focus on highlighting fallacies and misconceptions, using modelling, and using real data and familiar contexts where some prior expectation or insight about the data may be explored.[12]

Implications:

Activities related to patterns can incorporate covariational reasoning by offering opportunities to coordinate a counting sequence with the pattern (perhaps using number cards)

It may be beneficial for students developing covariational reasoning to focus on how quantities may change over time

Exploring covariation through multiple representations is important; students may have more success with identifying associations in graphs than in tables

Useful activities might include highlighting fallacies and misconceptions, using modelling, and using real data and/or familiar contexts

Students may find it easier to make judgements about relationships in familiar contexts, which can lead to useful cognitive conflicts

4 Using technology tools (rather than drawing on paper) to graph patterns, relationships and functions may support a global and dynamic view of patterns of covariation rather than a static view.[5] Using *multivariate* data sets (that have many different variables, preferably of different types) allows opportunities for students to explore relationships between multiple pairs of variables and test their predictions. Technology tools allow rapid drawing of graphs and trend lines to support this approach, often known as exploratory data analysis.[13,14] These tools may also allow data to be presented in non-standard formats that are easier to interpret for some students; for example, plotting one variable in the data as a dot plot, with a colour gradient imposed on the dots to give a sense of the value of the other variable.[15]

Implications:

Software tools can be used to support students' exploration of covariation in multivariate data through exploratory data analysis

Rapid drawing of graphs using technology can allow more time and focus on seeking and characterising patterns of covariation

Creating and exploring non-standard representations of data may support students in developing insight into associations between covarying variables

> "The more that you say, the less I know"
>
> **Taylor Swift, 2020**[16]

> "Imagine you're at a party, and you notice your friend Sarah laughing at all of John's jokes. You might start to wonder if there's a connection between John's humor and Sarah's amusement. That, my friend, is covariation in action – the tendency of two variables to change together"
>
> **NeuroLaunch, 2024**[17]

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