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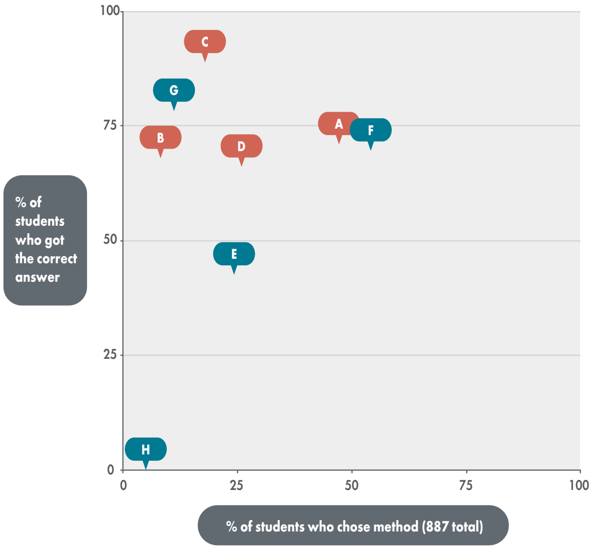
TALKING POINT:

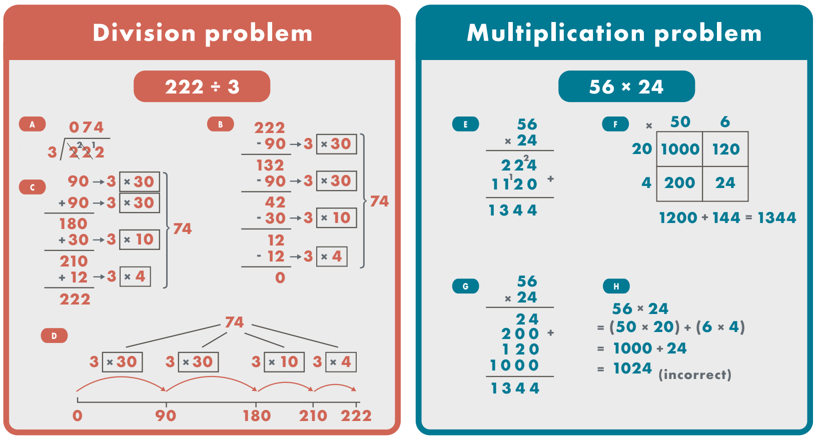
WHAT DOES RESEARCH SUGGEST ABOUT THE TEACHING AND LEARNING OF DIVISION AND MULTIPLICATION?

IN SUMMARY

- Consideration needs to be given to the ways in which multiplicative reasoning is different from additive reasoning
- Students' conceptual understanding of multiplicative reasoning is supported by composing and decomposing numbers through ideas of splitting, scaling and replicating
- The array is a particularly useful representation of division and multiplication as it captures two distinct inputs and can reveal commutative and distributive properties
- Exploring a variety of calculation strategies can support students in solving problems flexibly; it follows that it is useful to support this with ways of assessing division and multiplication that allow students to show their flexible knowledge of procedures and ability to choose strategies
- "Chunking" strategies for division (see infographic B and C) support students' mental methods
- It is suggested that students explore the concept of leftovers or remainders from the outset of their learning around concepts of division
- Developing students' thinking through the use of the array, an area model and the grid method for multiplication supports understanding of the two-dimensional structure of multiplication

Comparing success and choice of division and multiplication strategies chosen by students





1

Multiplicative reasoning (working both within and between quantities using processes of division and multiplication) is central to many aspects of mathematical learning and in employment and everyday life.[2] Moving from additive (based on counting structures) to multiplicative reasoning requires "a significant qualitative change in children's thinking."[3(p144)] For students to develop multiplicative reasoning, it is important to design tasks, activities and examples to help them distinguish additive reasoning from multiplicative reasoning, and in particular visualise concepts of multiplication beyond "just" repeated addition.[4] Features of additive reasoning tasks or situations are the joining of sets and only one type of input. Features of multiplicative reasoning tasks or situations are replication, two distinct and different inputs, distributivity, scaling, and an underlying ratio structure with four elements.[4&5]

IMPLICATIONS: Multiplicative reasoning is foundational to many mathematical, workplace and real-life ideas, and is a rich set of concepts that consist of more than just repeated addition. It is useful to consider the ways in which multiplicative reasoning is different from additive reasoning, comparing visualisations of each

Teachers might find it helpful to see multiplicative reasoning as (among others) replication, scaling, using two different types of input and involving a ratio structure with four elements

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2

Students benefit from representing division and multiplication by exploring ways of composing and decomposing numbers in different ways, including splitting, scaling and replicating.[5] In particular, using arrays can support these explorations of both division and multiplication and has several benefits: it reinforces the two-dimensionality of multiplication in contrast to additive reasoning, which is one-dimensional;[6] and it encourages a "visual demonstration of the commutative and distributive properties of multiplication."[Van de Walle et al., cited in 7(p311)]

IMPLICATIONS: Students' conceptual understanding of dividing and multiplying is supported by composing and decomposing numbers in different ways

Using arrays to represent multiplication and division helps demonstrate both distinctness of the two inputs being considered and the way in which multiplication is distributive and commutative

3

Researchers agree that developing automaticity in some multiplication and division facts is desirable as it frees up cognitive capacity for problem solving.[8] However, testing times tables, especially in timed conditions, may contribute to maths anxiety.[9] Since students' knowledge of multiple procedures and their ability to choose flexibly among them to solve problems is positively related to conceptual knowledge, assessing this may be a more useful representation of their multiplicative reasoning.[10]

IMPLICATIONS: There are benefits to timed tests as a route to automaticity with times tables for students, but they may also contribute to maths anxiety

It may be useful to consider ways of assessing times tables that allow students to show their flexible knowledge of procedures and ability to choose strategies

4

Research suggests that children experience more difficulty with the standard algorithm for division (see infographic method E) than with any other of the algorithms for basic operations.[11&12] An alternative to the standard algorithm, "chunking" strategies (see infographic B and C) are suggested as the key to successful mental methods.[13] "Chunking up" strategies (e.g., infographic C) have been found to be more reliable than "chunking down" strategies (e.g., infographic B).[1&14] Making sense of leftovers or remainders is important in division situations and so providing early opportunities to explore contextualised, "messy" division problems is recommended.[15]

IMPLICATIONS: The standard algorithm for division can be particularly problematic for students; exploring other strategies, particularly those based on chunking, may be helpful

Chunking strategies for division (see infographic B and C) are suggested as useful in supporting mental methods of calculation

Making sense of remainders and interpreting them in the problem context is important for all students

5

There are many different strategies associated with the concept of multiplication and it is suggested that an appreciation of a variety of these and the opportunity to compare them supports a more flexible approach to students' problem solving.[16] Exploring an array structure, developing this into an area model and then into a grid method provides opportunities for students to see and use structures that reveal commutative and distributive properties, which can relate to effective mental calculation and methods of estimation,[17&18] supporting these comparisons and choices.

IMPLICATIONS: It is recommended that students experience a variety of methods of multiplication in order to support their selection of appropriate problem-solving strategies

Progressing from an array through an area model to a grid method supports conceptual understanding of multiplication and aids mental calculation and estimation

"Riding my bicycle gets me to my office in about the same time as taking my car, but the two processes are very different"

Devlin, 2008[19]

"When I taught A-level Maths ... it was not uncommon to find students who ... did not appreciate that multiplying by ½ was equivalent to dividing by 2"

Stripp, 2015[20]

Fran Watson, Lucy Rycroft-Smith & Tabitha Gould, 2022

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