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CAMBRIDGE √Mathematics

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TALKING POINT:

WHAT DOES RESEARCH SUGGEST ABOUT THE TEACHING AND LEARNING OF SIMILARITY?

IN SUMMARY

- Similarity brings proportional reasoning and geometry together, offering a context through which to develop the idea of mathematical argument and proof and move from visual to logical reasoning
- Early intuitions about similarity are developed when children play with and describe models and distorted 2D and 3D shapes
- The everyday language and ideas of sameness compared to that of mathematical similarity can cause confusion for students
- Static approaches to similarity involve identifying scale factors and "matching" lengths or angles, although identifying corresponding parts is often challenging for students
- Similarity tasks can be considered as differentiating between similar and non-similar shapes or constructing similar shapes
- Similarity can be explored through the additive process of tiling or the multiplicative idea of scaling; both give insights into the geometrical properties of the shapes or objects involved
- Similar shapes or objects can be identified, and missing values calculated, by considering between ratios, within ratios and scale factors, or using more dynamic approaches involving technology

Four ways to approach finding missing values when working with similarity

Ayen is making a larger version of this flag of the Republic of the Sudan to hang on the classroom wall.

They have cut a big piece of paper so it is 210cm wide.

What will the side of the big green triangle measure?

METHOD 1: Between ratio

$$\frac{7.2}{210} = \frac{3}{x}$$

Corresponding lengths go in the same fraction

$$x = \frac{3 \times 210}{7.2}$$

$$x = 87.5\text{cm}$$

METHOD 2: Within ratio

Smaller shape → $\frac{3}{7.2} = \frac{x}{210}$ ← Bigger shape

$$\frac{3 \times 210}{7.2} = x$$

$$x = 87.5\text{cm}$$

METHOD 3: Scale factor

$$7.2 : 210$$

$$\text{Scale factor} = \frac{210}{7.2}$$

$$= 29\tfrac{1}{6}$$

$$x = 3 \times 29\tfrac{1}{6}$$

$$x = 87.5\text{cm}$$

METHOD 4: Transformation-based

Enlarging small flag to make bigger flag as instances within a class of similiar figures

1

Similarity can be defined from a variety of perspectives: for example, similar shapes are the same shape yet different sizes; corresponding angles are equal; the lengths of corresponding edges are connected by a common ratio, as are corresponding areas and volumes (where appropriate). Similarity brings together geometric, spatial and numerical reasoning.[3] With carefully designed tasks, similarity offers a context in which geometric constructions can encourage students to develop mathematical arguments, reasoning and proof, as well as form conjectures and consider ideas of validity.[4] In these situations, dynamic geometry environments may support a move from visual and intuitive reasoning to logical-deductive reasoning – using accepted facts to form a logical argument for the truth of a new statement.[5]

IMPLICATIONS: There are several perspectives from which to define similarity

Similarity brings together geometry and proportion, and offers a context in which to conjecture, reason, argue and prove

Dynamic geometry environments may support students to move from visual to logical reasoning

2

Young children build early ideas of similarity when playing with physical models, both in and out of school contexts.[6] The distinction between sameness and similarity poses a challenge and research suggests students should visually explore distorted 2D shapes and 3D objects[7] in order to develop informal language to describe how they compare, including vocabulary such as "thinner" and "pointier."[8] These early visual ideas are gradually replaced by numerical comparisons and this process can be supported by estimating the size of objects in photos with respect to known measure.[8] Ideas are then refined as students visually scan (to explore characteristics and properties), reproduce, or scale up images, although students may initially rely on additive strategies.[9]

IMPLICATIONS: Early intuitions about similarity are developed when children play with models

Ideas about similarity are encountered and developed when exploring 2D and 2D shapes, especially distorted shapes, and using informal language of comparison

The language of "same" and "similar" needs careful handling when applied in the mathematics classroom

3

Geometrically similar shapes can be seen in three ways: as the same shape but not necessarily the same size (enlargements or dilations); shapes where corresponding lengths are in proportion and corresponding angles of equal size; or shapes where a combination of translations, reflections, rotations and enlargements transform one shape into the other.[9] These first two static approaches may include proportional relationships between and/or within similar figures as well as scale factors,[10] but these static perspectives rely upon identifying corresponding parts of the figures which poses a significant challenge to many students.[11] Similarity can be explored through the additive process of tiling or the multiplicative idea of scaling; both give insights into the geometrical properties of the shapes or objects involved.[8] Dynamic geometry environments can support an alternative, dynamic approach, using transformations, although research concerning teachers using a DGE in such situations is sparse.[9]

IMPLICATIONS: Static approaches to similarity involve identifying scale factors and considering proportions by looking at, for example, lengths and angles

Identifying corresponding parts of figures is often a significant challenge for students

Students can explore additive, tiling approaches and multiplicative, scaling approaches to similarity to help support an understanding of proportional reasoning

Using a dynamic geometry environment to explore how to transform one shape into another may support deeper, dynamic understanding of ideas about similarity

4

Similarity tasks and problems can be classified as either *differentiating* or *constructing*.[8] Differentiating similar from non-similar shapes initially relies on visual factors such as the general appearance and position. Later, students looking for proof of a lack of similarity may compare angles or lengths, look for qualitative or quantitative relationships between the two figures, or investigate the ability to tile one to form the other or enlarge one to another. By contrast, construction-type similarity tasks involve not just the construction of similar figures but also "missing value" type tasks, where students may identify these values by using *between ratios, within ratios* and/or scale factors (see infographic).

IMPLICATIONS: Similarity tasks can be considered as differentiating between similar and non-similar shapes or constructing similar shapes

Constructing similar shapes includes "missing value" type problems

Students can use ratios between shapes, within shapes, or scale factors to identify missing values

"With the latest James Bond thriller ... the film's producers have ... [created a] two-thirds scale replica of Bond's iconic Aston Martin DB5 – the seminal Bond car... It's an absolute bargain at just £90,000, which is a lot less than the million dollars plus you can expect to pay for a full-size DB5"

Butler (2001)[1]

"Many phenomena in the world around us are determined by the interplay between the growth relationships that obtain [sic] in different dimensions. This interplay explains why the largest animals in the world live in the ocean and why trees need leaves"

Chazan (1988)[2(p38)]

Rachael Horsman & Lucy Rycroft-Smith, 2021

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