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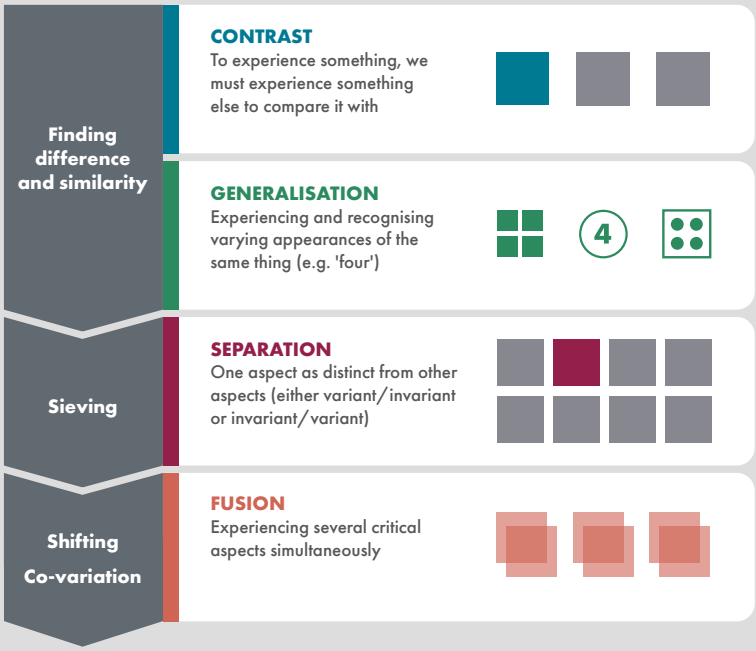
TALKING POINT:

HOW MIGHT VARIATION THEORY AFFECT MATHEMATICS TEACHING AND LEARNING?

IN SUMMARY

- Variation theory (VT) is a way of analysing and planning teaching and learning activities which focuses on what changes and what stays the same and the effect this might have
- Researchers suggest different emphases for different critical aspects when structuring variation in maths lessons, but recommend that students should see difference before sameness, including counter- or non-examples
- Well-structured tasks are important, but successful use of VT also requires teachers to be effective in using these tasks to direct pupil attention
- Using VT can be a way of helping maths teachers develop professionally, but may also formally characterise what they are already doing
- Teachers should make sure they select and use examples very carefully as a way of drawing attention to mathematical structure
- Professional development that increases maths teacher expertise in example and problem choice may be particularly beneficial

Patterns of variation



Adapted from Marton & Tsui (2004) and Leung (2013)

1

Variation theory (VT) is not unique to mathematics education but has often been used to explore mathematics *learning* by focusing on the different ways that it is possible to look at, or experience, the same thing;[2] or mathematics *teaching* by paying 'deliberate attention to what and how [teachers] exemplify mathematical ideas, processes and practice in their teaching'.[3] It assumes that 'learning is about developing new ways of seeing or experiencing the object of learning' and so it is necessary to 'experience certain patterns of variation and invariance in order to discern **critical features** [key ideas] of the object of learning'.[4] It has been described as an 'analytical tool' with which to understand, reflect upon and plan maths teaching and learning episodes.[5] Applying VT to mathematical task design and planning requires identifying the critical feature (also known as critical aspect or the intended object of learning) and varying this against an invariant background (or vice versa) to direct learners' attention towards it, as well as anticipating their reactions and incorporating this into teaching plans.[6]

IMPLICATIONS: Teachers can use VT to analyse maths teaching or learning as it suggests (a particular type of) planned variation is necessary for students to notice what is to be learned

Considering lesson activities using VT principles can allow teachers to direct students' attention and anticipate their responses

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It is important to note that different researchers suggest different emphases in terms of exactly what to vary and what to keep invariant in structured task design and 'variation usage in structured exercises varies considerably from country to country and from text to text.'[6] Some VT research suggests that learners should experience difference before sameness; e.g. by looking at a series of examples that 'share the aimed-at-meaning but differ otherwise', or considering counter-examples, allowing learners to *contrast*.[7] The suggested next stages – *generalisation*, *separation*, and *fusion* – can be seen on the infographic.[8] However, 'well-designed tasks and examples...are usually insufficient for learning...it is important that teaching draws attention to these patterns.'[6]

IMPLICATIONS: Teachers should recognise that researchers suggest different emphases for different critical aspects in the way that lesson materials are structured and enacted

Research suggests that learners should experience difference before sameness, including counter- or non-examples

Well-structured tasks are not enough to enact VT successfully – teachers must also be effective in directing students' attention to the intended critical feature/s

3

It has been suggested that one way using VT can help teachers is to 'facilitate teachers' awareness of mathematics as a connected field of study by directing their attention to structural similarities and differences'.[9] Teachers often characterise elements of VT as things they already do, or 'common sense'.[2] However, some studies have shown that maths teachers are often unconscious of differences in quality in the example choices they make or that they make them randomly.[6] VT encourages a focus on example choice as of 'decisive importance';[6] choosing multiple examples carefully to allow learners to distinguish between essential as opposed to non-essential aspects may enhance learners' perception of 'deep structures'[10] and can improve student outcomes.[6,13]

IMPLICATIONS: VT can help teachers' professional development in terms of awareness of mathematical connections, but may also be a theory that simply formally characterises what they do already

Teachers should ensure they select (collections of) examples carefully, considering their utility in revealing mathematical structure

4

VT suggests that it is important for learners to avoid focusing on surface features but to examine mathematical structure[8] which supports the idea of teaching for deep understanding, regarded as crucially important in successful mathematics learning.[e.g.11] It has been suggested that the so-called 'Chinese paradoxical phenomenon' – where Chinese students are often successful in mathematics in comparison to students in other countries despite a perceived focus on rote learning and memorisation – may be due to teachers' expertise in 'juxtaposing problems and examples' and anticipating student responses in their planning using VT.[12]

IMPLICATIONS: VT may help learners explore mathematical structure and support deep learning in mathematics

Professional development that increases teacher expertise in carefully structured example and problem choice in maths lessons may be particularly beneficial

'Tasks that carefully display constrained variation are generally likely to result in progress in ways that unstructured sets of tasks do not, as long as learners are working within mathematically supportive learning environments'[1]

Watson and Mason, 2008

'Variation theory has several dimensions, including use of multiple representations of what a concept is, and what it is not'

National Centre for Excellence in Teaching Mathematics, UK

Lucy Rycroft-Smith, 2019

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