Transcript for CoffeePod, based on Espresso 35

Welcome!

This is an audio recording of Espresso 35, which was published in February 2021 and written by Lucy Rycroft-Smith and Tabitha Gould.

Espressos are 2-page filtered research documents from Cambridge Mathematics. Each one focuses on selecting and summarising research on a given area of mathematics education, called a Talking Point. They are published as free pdf documents on the Cambridge Mathematics website www.cambridgemaths.org.

This CoffeePod is 29 minutes and 29 seconds long in total. It comprises the following sections: a talking point; quotes; a summary; the main text of the Espresso; the infographic; and an optional reference section which is also available as a separate document linked in the video notes.

You will hear the following sound to indicate a reference [tone], followed by the reference number to help you identify it.

The chapter timings for the different sections, and a link to the original published Espresso, can also be found in the video notes.

You can give us feedback or ask questions in the comments section under this video – we'd love to hear from you!

Talking point:

What does research suggest about the number line?

Here are two quotes from the research that you might find interesting to think about:

"The number line—a conceptual tool that allows for numbers to be conceived as locations along a line mapping numerical differences onto differences in spatial extension… is mathematically simple, yet it is extraordinarily powerful"[1]

"The empty number line [is] a very powerful tool for enhancing communication in the classroom"[16]

In summary:

We found seven main implications for teachers in the research. We have numbered them here for ease of listening but they are not intended to be hierarchical (that is, no one implication is more important than any of the others). The seven implications are:

1. A number line is a (usually linear) spatial representation of number which helps support mathematical understanding and connections between measure, number and statistics
2. Using a number line representation, which is likely to be innate for young children, may form an important part of understanding place value and proportional reasoning
3. Number line estimation tasks are suggested as an efficient way to assess broader mathematics competence
4. Young children can use bead strings or linear number tracks as early number lines, later moving to notched number lines
5. Use of an empty number line model is suggested, as it is flexible, allows for student methods and errors to be seen easily, and is supportive of a wide range of methods
6. Training students to use and construct number line models based on their 'innate' mental number lines supports mathematical development, and the use of the empty number line can promote conceptual understanding
7. Double number lines can help support proportional reasoning, especially highlighting the idea of multiplication as scaling

Here is more detail about the research which fed into the summary implications. It is organised into four paragraphs which each have a set of implications for teachers at the end.

Paragraph one:

The idea of the number line may refer to various different models of placing numbers on (or reading them from) a line, a length of track, a scale, or a piece of string. It is a linear spatial representation of number that is considered to be an important model in supporting and developing connections between work in measure, number and statistics – in particular when sorting quantity and data into space on the Cartesian plane[2,3] – as well as moving from arithmetic to algebraic structure.[4] Using the number line successfully is associated with a developing understanding of place value and in particular of proportion,[5] because it is a representation that allows reasoning about continuous quantities.[6] Use of the number line throughout a student's mathematical career can help develop their conceptions of the density of number from counting numbers, to integers, to rational numbers and irrational numbers.[7]

There are two implications of paragraph one:

First implication: The number line is a model that helps support mathematical understanding and connections between measure, number and statistics

Second implication: Using a number line helps support the development of place value and proportional reasoning

Paragraph two:

Research suggests that children possess an innate sense of a number line whose orientation (whether left to right or right to left for smaller to larger numbers) may be related to the direction of reading they have learned.[8] Researchers currently do not agree whether this early mental number line is linear (equally spaced) or logarithmic (where the numbers get closer together as they get larger) or a piecemeal composite structure.[9] Performance on a simple number line estimation task has been found to correlate strongly with performance on counting, arithmetic, and standardised school achievement tests, and can therefore be used as a good proxy for measuring broader mathematical competences which do not need prior knowledge.[10] Using a number line model successfully has been described as making use of this innate mental structure to develop it into a tool to reason with; a shift from just a *model of*, to also a *model for*.[11]

There are two implications of paragraph two:

First implication: Children seem to have a mental conception of a number line structure that is likely to be left to right, smallest to largest, if they have learned to read left to right, and vice versa

Second implication: Number line estimation tasks are suggested as an efficient way to assess broader mathematics competences

Paragraph three:

Young children can use coloured beads on a string or other abacus-type structures, emphasising fives and tens, to support spatial representations of number.[12] Moving from this kind of representation to that of a number line (with labelled points or notches) is challenging[13] and should be developed in parallel with developing ideas about pacing off length. Later, working on a structured line which has some marked numbers provided, may give way to the use of the empty number line. This has no numbers or markers; these are added by students. The empty number line is recommended because it is a model for number operations that easily allows methods and errors to be visible.[14] As a model, it is open to informal strategies while supporting the development of more formal and efficient strategies; it also enhances the flexibility of mental strategies,[15] and may be important in allowing student autonomy as it can be used to record strategies rather than prescribing them.[16]

There are three implications of paragraph three:

First implication: Young children's work on the spatial representation of number can be supported with structured bead strings or linear number tracks

Second implication: Moving students from number tracks to number lines, with labelled notches rather than spaces, is important but may be challenging for them

Third implication: Use of an empty number line model is suggested because it allows student methods and errors to be seen easily, is flexible, and supports student autonomy in choice of methods

Paragraph four:

There is evidence that training students to use the number line more competently may support mathematical development, especially in arithmetic and particularly through the use of number tracks in games; however, this is only when the number tracks are linear, not when they are circular.[17] Construction of a linear number line allowed students at age 6 to solve a wide range of maths problems more successfully.[18] The use of the double number line helps extend the metaphor of multiplication as stretching, as well as helping students understand that quantities continue to extend beyond the 'whole' unit identified (for example, that it can be meaningful to find 125%).[19] Using empty number lines, particularly as an additive reasoning tool at first,[20] can promote conceptual understanding and reduce reliance on counting strategies, but teacher competence is an important mediating factor[21] and students need clear, explicit teaching on how to use them.[22]

There are three implications of paragraph four:

First implication: Training students to use and construct number line models supports mathematical development; linear number tracks in games are better for this than circular ones

Second implication: Double number lines can help support proportional reasoning, especially highlighting the idea of multiplication as scaling

Third implication: The use of the empty number line, in particular for additive reasoning, can promote conceptual understanding, provided teachers support students in using it

The Espresso also has an infographic

The title of the infographic is 'The number line: a flexible and useful model'. It appears as seven diagrams going down the page, each labelled with a title, to represent using the number line in different and perhaps increasingly complex ways.

The first diagram on the infographic is labelled 'Bead string / abacus'; it shows an image of three strings of beads arranged vertically. The top string is threaded (from left to right) with four white beads, a space, and then another white bead. The middle string is threaded (from left to right) with five white beads next to two red beads, then a space, then three more red beads. The bottom string is threaded (from left to right) with five white beads, then a space, then five red beads.

The second diagram on the infographic is labelled 'Number track'; it shows a pleated or wiggly track with labelled spaces. The track appears torn off or incomplete at both ends. There are ten labelled spaces, each with the digits of the numbers 0 to 9 consecutively inside the spaces, from left to right. The incomplete first and last spaces on the track are unlabelled.

The third diagram on the infographic is labelled 'Structured number line'; it shows a straight horizontal line crossed with other smaller perpendicular lines, or notches, dividing it into equal parts. Each notch is labelled below the number line with the numbers 0 to 12 consecutively, left to right.

The fourth diagram on the infographic is labelled 'Wrapped number line'; it shows a circle with notches around it, similar to the previous diagram but this time the line has been 'wrapped' into a circle by joining the beginning and the end together so it resembles a clock face. It is also labelled with consecutive numbers on the notches but this time with the numbers 1 to 12 clockwise, with the very top labelled with the numbers 12 and 0.

The fifth diagram on the infographic is labelled 'Empty number line (with examples added)'; it shows a straight horizontal line with five notches on it, with differing distances between them. The first notch on the left is labelled -3 above the line and the last notch on the right is labelled 22 above the

line. Below the line curved marks between the notches show the distances in units; between the first and second notches, labelled 3 in digits; between the second and fourth notches, labelled 20 in digits; and between the fourth and fifth notches, labelled 2 in digits. The third notch is exactly in the middle of the distance labelled 20. The line continues past the notches both at the beginning and at the end.

The sixth diagram on the infographic is labelled ‘Double number line’; it again shows a straight, horizontal line with small vertical notches across it. This time, the three notches, which are different distances apart, are labelled both above the line and below it; the first notch is labelled above with 0 and below with 0%, the second notch is labelled above with 400 in figures and with 100% below, and the third notch is labelled above with 500 in figures and with 125% below.

The seventh and final diagram on the infographic is labelled ‘Variable number line’; it is very similar to the previous diagram with three notches different distances apart on a straight horizontal line. The notches only have one label below each and left to right are labelled as ‘n – 3’, ‘n’ and ‘n + 2’.

Thanks for sampling this new product.

We loved you joining us.

The details of the references are coming next, so feel free to listen or stop here – whichever you prefer. If you enjoyed this CoffeePod, there are others available, and more recordings in progress.

We’d love to hear your thoughts about this CoffeePod. Please email us at admin@cambridgemaths.org. And you can also read the original Espresso by going to www.cambridgemaths.org.

The Espresso has a reference list of 22 entries written using APA 7 notation.

The references can either be accessed as a separate document or in the published Espresso via links in the video information box. Alternatively, you can continue to listen to them here.

Reference 1: Núñez, R. (2016). How much mathematics is “hardwired,” if any at all: Biological evolution, development, and the essential role of culture. In M. D. Sera, S. M. Carlson & M. Maratsos (Eds.), *Minnesota symposium on child psychology* (Vol. 38, pp. 83–124). John Wiley & Sons.

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Reference 3: Metz, M. L. (2010). Using GAISE and NCTM standards as frameworks for teaching probability and statistics to pre-service elementary and middle school mathematics teachers. *Journal of Statistics Education, 18*(3), 1–27.

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Reference 6: Moss, J., & Case, R. (1999). Developing children's understanding of the rational numbers: A new model and an experimental curriculum. *Journal for Research in Mathematics Education, 30*(2), 122–147.

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Reference 9: Siegler, R. S., & Booth, J. L. (2004). Development of numerical estimation in young children. *Child Development, 75*(2), 428–444.

Reference 10: Schneider, M., Merz, S., Stricker, J., De Smedt, B., Torbeyns, J., Verschaffel, L., & Luwel, K. (2018). Associations of number line estimation with mathematical competence: A meta-analysis. *Child Development, 89*(5), 1467–1484.

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Reference 12: Bobis, J., & Bobis, E. (2005). The empty number line: Making children's thinking visible. In M. Coupland, J. Anderson & T. Spencer (Eds.), *Making mathematics vital: Proceedings of the twentieth biennial conference of the Australian Association of Mathematics Teachers* (pp. 66–72). Australian Association of Mathematics Teachers.

Reference 13: Askew, M. (2016). *A practical guide to transforming primary mathematics: Activities and tasks that really work*. Routledge.

Reference 14: Bramald, R. (2000). Introducing the empty number line: The Dutch approach to teaching number skills. *Education 3-13, 28*(3), 5–12.

Reference 15: Beishuizen, M. (2010). The empty number line. In I. Thompson (Ed.), *Issues in teaching numeracy in primary schools* (2nd ed., pp. 174–187). McGraw-Hill Education.

Reference 16: Bobis, J. (2007). The empty number line: A useful tool or just another procedure? *Teaching Children Mathematics, 13*(8), 410–413.

Reference 17: Siegler, R. S., & Ramani, G. B. (2009). Playing linear number board games – but not circular ones – improves low-income preschoolers' numerical understanding. *Journal of Educational Psychology, 101*(3), 545–560.

Reference 18: Case, R., Okamoto, Y., Griffin, S., McKeough, A., Bleiker, C., Henderson, B., Stephenson, K. M., Siegler, R. S., & Keating, D. P. (1996). The role of central conceptual structures in the development of children's thought. *Monographs of the Society for Research in Child Development, 61*(1/2), i–295.

Reference 19: Orrill, C. H., & Brown, R. E. (2012). Making sense of double number lines in professional development: Exploring teachers' understandings of proportional relationships. *Journal of Mathematics Teacher Education, 15*, 381–403.

Reference 20: van den Heuvel-Panhuizen, M. (Ed.). (2001). *Children learn mathematics: A learning-teaching trajectory with intermediate attainment targets for calculation with whole numbers in primary school*. Sense Publishers.

Reference 21: Kaufmann, S., & Wessolowski, S. (2014). *Rechenstörungen. Diagnose und Förderbausteine [Mathematical learning disabilities. Diagnosis and building blocks of intervention]*. Klett/Kallmeyer.

Reference 22: Sarama, J., & Clements, D. H. (2009). *Early childhood mathematics education research: Learning trajectories for young children*. Routledge.