Cambridge Mathematics Framework

Draft 1 | March 2015

The mission of the University of Cambridge is to contribute to society through the pursuit of education, learning and research at the highest international levels of excellence.

Cambridge Mathematics is a collaborative enterprise in pursuit of this mission. The four University partners[1] are committed to championing and securing a world-class mathematics education for all students from 5 – 19 year olds, applicable to both national and international contexts and based on evidence from research and practice. We will call on our institutional courage and use the leadership, authority, resources and expertise of the University to develop a coherent, transparent, evidence-based vision for Cambridge Mathematics – a vision that will make a major contribution to mathematics education internationally.

A MANIFESTO FOR CAMBRIDGE MATHEMATICS

We believe that the study of mathematics is worthwhile for its own sake and as a key form of intellectual development. It is critical for the understanding of many other subjects and essential for functioning within modern society. Our vision is that the model of mathematics education that we intend to devise will emphasise the richness and power of mathematics, will encourage continued study of the subject and will be recognised world-wide as innovative and rigorous in approach. It will take into account the different needs in a changing world and contribute to learners' personal, societal and economic wellbeing. The enactment of the vision will draw on the expertise that resides in the University, and on that of our colleagues across the world.

THE AIMS OF CAMBRIDGE MATHEMATICS

Cambridge Mathematics is a long-term programme of developments and by 2020 we aim to have made considerable progress towards the seven subsidiary aims:

- to champion and secure **access** to a quality mathematics education for all
- to **collaborate** to use our position in maths education, to show **leadership** and to develop an authoritative **voice**
- to develop a coherent Cambridge Mathematics **Framework** for all ages and types of learner with a strong distinctive approach, **led by academics and educationalists** and supported by a strong **research base**
- to develop and make available world-class **teaching and learning materials**
- to support an infrastructure to enhance the quality of **teacher education and continuing professional development**
- to develop **assessments** that support the development of powerful mathematical reasoning
- to develop an approach that is **recognised and valued** by parents, young people, teachers, institutions and governments.

THE PRINCIPLES OF CAMBRIDGE MATHEMATICS

The four principles underpinning our work are:

- access for all
- evidence based
- collaboration and consultation
- coherent and integrated programme

and these principles accord with the University's mission statement.

THE FOUR ELEMENTS OF CAMBRIDGE MATHEMATICS

The **Cambridge Mathematics Framework** is the first piece of work towards the Cambridge Mathematics vision. It is the strategic document which will inform all the subsequent elements of Cambridge Mathematics, and so it is essential that it is as well constructed as it can possibly be. The 'launch' in March 2015 is not the launch of a finished product but the presentation of initial ideas and their provenance, a proof of concept, together with an invitation to knowledgeable and interested colleagues, within and outside the University, here and abroad, to join in the conversation to test and refine this first draft. Once the Framework is fully populated, our attention will turn to **assessments**, **resources** and **professional development**, all of which together will make up an integrated and coherent whole. So, in addition, the consultation offers an opportunity to do some blue skies thinking about the future development of the other elements of Cambridge Mathematics.

THE FIRST SHARED VERSION OF THE FRAMEWORK

There already exist frameworks for mathematics. We will build on progress already made, make explicit the problematic issues, and interrogate new and relevant research, for example on how students learn, the impact of technology, the advances in mathematics itself. We want to open up the debate about the nature of mathematics education, and the nature of the maths we choose to communicate to the next generation. We want the Framework to be a seminal concept that will inform both curricula and assessment and, later, resource design and professional development. This is not a trivial ambition and there will, we hope, be many deep and varied discussions and debates along the way. Our ultimate aim is that eventually the Framework will be a coherent and flexible presentation of the mathematics with which students aged 5 to 19 could be reasonably expected to engage.

The development of this initial version has been informed mostly[2] by international comparisons and, time permitting, research evidence. There is a lot more thinking to be done about the structure before we even start to consider the content. We are not sure that we have the dimensions correctly identified, or even if a multidimensional approach is the best. However, one has to start somewhere, so in this initial version we have chosen to present the **content domain** as three interrelated areas rather than as distinct subdomains. A second dimension is the **progression in this content** which we exemplify in two parallel ways: a) threshold skills and procedures and b) initial exposure to mathematical concepts.[3] A third dimension describes the depth of conceptual understanding and, since we believe that problem solving is at the heart of studying mathematics, we include, as a fourth dimension, our current model of the mathematical processes that make up mathematical thinking and problem solving.

Allied to the dimensions we think there are other key factors, such as the context in which the mathematics is practised, and the ways in which technology can be used to enhance the learning of mathematics. These will, we predict, have a greater or lesser importance depending on the purpose of the curriculum and assessment.

In summary the Framework will:

- be evidenced both from the study of a range of mathematics curricula and mathematics assessments, and from a theoretical perspective of conceptual progression
- be illustrated by indicative content and exemplar performances
- allow for the description of skills and dispositions necessary for effective use of mathematics.

Below we explore the dimensions in more detail.

THE SUBDOMAIN DIMENSION:

Internationally there is a high degree of coherence on what areas of content should be learned, although the way the mathematics is described and grouped varies from country to country.[4] We tried firstly to use the four content categories which will be used in PISA 2015: *Quantity, Space and shape, Change and relationships, Uncertainty and Data handling*.[5] However the PISA framework is designed to inform the assessment of Mathematical Literacy (and for a particular stage) and we felt that our first attempt at mapping content to this was unsatisfactory. Our subsequent mapping uses the relatively common subdomains of: *Number, operations and algebra relations; Geometry and measurement; Data and uncertainty*: we will review these again once we also include content that is traditionally met post –16. However, whichever way the mathematical content is cut, most mathematical ideas do not fit neatly into one category and since we believe this is an important factor in understanding the mathematical domain, we have illustrated this interconnectedness in the Framework by italics.

THE PROGRESSION IN CONTENT DIMENSION:

We envisage that the Framework will support the mapping of curricula and assessments for a range of purposes – from basic numeracy through to an academic route for those intending to pursue higher level study. Any progression dimension therefore can be only loosely age related. We have chosen initially to follow the Common European Framework[6] structure of three stages each divided into two. A learner between the ages of 5 and 11 would usually be working within Stage 1, although this stage may also be suitable for those studying an adult numeracy curriculum, for example. Learners between 11 and 16 would normally be working within Stage 2 and older learners within Stage 3. An international comparison[7] of school systems indicates this would be broadly in line with most, although the age at which learners move through the school system do vary.

Within each stage, and for each subdomain, the mathematics we have chosen to include is described in two ways – for summative assessment purposes by *threshold skills or procedures*[8] and for curriculum design purposes by a progression in exposure to, or introduction of, *key mathematical concepts*.

The use of threshold skills or procedures is well documented as a structure for assessment design.[9] There are some sequencing decisions that appear to be currently and commonly agreed, although there are some exceptions to the rule.[10] At this stage we have chosen to make the content very spare because we believe that the more detail is included, the less scope there will be for the development of new, more innovative, and more effective sequencing models and pedagogies, for example where the affordances of technology may impact on the sequence of learning.

Curriculum frameworks sometimes use the idea of threshold concepts[11] – those which, unless they are effectively grasped, will act as a barrier to progress. We have instead decided to use the idea of the *initial* introduction of a concept, rather than identify by what stage we think a concept has been secured. We assume that, once 'primed', the concept will be refined over and over again in new contexts as the learner moves through the mathematics. We welcome feedback on the practicality and advantages/disadvantages of this idea.

There are many alternative formats that we could consider, for example replacing stages with a continuous progression. However this would require far more detail at this stage and, although it could be more useful in a formative way would, we predict, be less so for the design of summative assessments. It would also make mapping of existing assessments less straight forward. We are very keen to hear of possible alternative visualisations that would also make good use of digital formats.

THE CONCEPTUAL UNDERSTANDING DIMENSION

A cursory view of the literature indicates different models of conceptual understanding, both generally and in the context of mathematical understanding. The well-known taxonomy attributed to Bloom[12] (Bloom et al, 1956) suggests six different levels at which this might be considered: knowledge, comprehension, application, analysis, evaluation and synthesis. In mathematics Richard Skemp[13] (1975) detected two different modes of understanding that people referred to when describing their understanding of mathematics: instrumental and relational. Skemp suggested that in simple terms instrumental understanding might be thought of as "rules without reasons" whereas relational understanding includes understanding not only of how to do something but also why it works. In other words relational understanding gives insight into mathematical structure allowing one to recreate rules where necessary.

More sophisticated models of mathematical understanding recognise and build on these ideas, for example, Pirie and Kieren's model of the growth of understanding (Pirie and Kieren, 1989)[14] suggests that starting from a stage of 'primitive knowing' understanding develops through seven further stages.

We were very attracted to the idea of APOS theory as developed by Dubinsky[15] (1991). He suggests four stages with the first of these *action* (A) corresponding in some ways to Skemp's notion of instrumental or procedural understanding, that is, the ability to carry out a rule. Understanding of "why" the rule works suggests working at the *process* level (P) with further development of understanding leading to the learner being able to work with the process as a new *object* (O). For example, given a linear function, f(x), at an *action* level a learner would be able to use this to develop a table of values. Operating at a *process* level the learner would understand the concept of linear function and be able to explain how varying the x multiplier or the added constant affects the values of f(x) over a given domain. At a more sophisticated level the learner will be able to use the function f(x) as an *object*, for example, combining it with another function g(x) to give h(x). Further understanding at the level of *schema* (S) provides insight into relationships between mathematical ideas. For example, how linear functions relate to functions that model situations where direct proportion holds, how the algebraic formulation of functions relates to how they can be considered using geometric transformations of basic functions and so on.

We consider this to be a very important dimension of learning mathematics and, having considered the various models above, we have amalgamated elements of them to provide three levels at which to consider mathematical activity:

Procedural fluency refers to when rules and procedures are applied without concern about their structure and why they work. For example, if asked to solve an algebraic equation such as $2(3x + 1) = 32$ this can be achieved by 'balancing' or flow charts.

Relational understanding refers to the understanding of mathematical structure and consequently why rules/methods work. For example, if asked to solve a pair of simultaneous linear equations the learner understands that the solution lies where a single pair of values of x and y satisfies both equations and is able to use this knowledge to make sense of methods that eliminate one of the variables to form a single equation in the other that can then be solved.

Schematic understanding is such that the learner has a sense of how mathematical actions, processes and objects are connected. Such understanding leads to the learner developing a mental image of mathematical connectivity.

THE WORKING MATHEMATICALLY DIMENSION

Mathematical activity might involve us in:

- representing
- visualising
- classifying
- ordering
- measuring and quantifying
- reasoning
- convincing
- proving
- interpreting
- evaluating

….and so on. The question of how to capture mathematical *practices* such as these is one that large scale studies such as PISA and TIMSS, curriculum and assessment designers, and others have dealt with in different ways. However, in common with such developments, we have taken problem solving as the term that captures the idea that at the heart of meaningful mathematical activity is a problem. Whilst we recognise the place and role of the practice of developing skills and procedural fluency as essential towards effectiveness in such activity, this major dimension of the Framework attempts to capture learners' engagement in important aspects of problem solving.

As Watson recognises, "The phrase 'problem-solving' has many meanings and the research literature often fails to make distinctions.".[16] Whilst we did not find the subdomains used in recent PISA writing helpful, we have built on the attempts of the PISA series of studies to provide insight into learners' mathematical literacy,[17] focusing on the verbs 'formulate,' 'employ,' and 'interpret' as the key three processes in which learners engage (OECD, 2015). The Framework develops these categories by redesignating 'employing' as 'analysing' and by additionally including 'communicating' as we believe this is an important aspect of working mathematically.

So the categories of the Working Mathematically dimension of the Framework we are working with currently are:

Formulating: this involves drawing on mathematical knowledge, skills and understanding to gain insight into the mathematical structure of a problem situation. It requires an awareness of what mathematics it might be useful to work with. The formulating stage may require a period of exploration, conjecturing and experimenting. Representing, visualising, systematising, and developing notation may all be typically helpful strategies and, when working in non-mathematical contexts, making sense of and structuring the situation by identifying constants and variables and making assumptions about these may all be necessary.

Analysing: we use the term analysing to characterise activity that involves applying mathematical knowledge, skills and understanding towards a mathematical solution, ie an umbrella term to attempt to capture this activity that is often considered as 'doing the maths'. In general, in this phase of problem solving, work is entirely in the world of mathematics and requires a range of facts, skills, logical-deductive thinking and understanding to carry out calculations, graphical work, algebraic manipulation and so on. In this phase cognitive engagement with mathematical constructs is required: for different learners this may require procedural, relational or schematic understanding at different times in the process. Outcomes remain mathematical.

Interpreting/evaluating: involves making sense of the outcomes of the mathematical work in relation to the problem that is being worked on. This requires considering whether the mathematising of the problem was adequate and if, and how, this might be improved. In light of considerations at this stage it may be necessary to engage in re-formulating the mathematical problem to provide a new mathematical structure and re-analysing this.

Communicating: "Because mathematics is so often conveyed in symbols, oral and written, communication about mathematical ideas is not always recognized as an important part of mathematics education." Cobb, Wood and Yackel (1994) We believe that being able to communicate mathematically is an important mathematical process: both to oneself when working on a problem and when presenting findings. It draws on knowledge of appropriate notation and use of conventions, and text that links mathematical statements and representations to provide an account of exactly how the situation has been analysed and how the resulting outcome relates to the original problem and the validity of the solution.

OTHER FACTORS

Context

We recognise that a major distinction needs to be drawn between solving a problem in the world of mathematics itself and solving a problem that arises in contexts outside of mathematics, for example, in other academic study, in personal or societal situations or in the world of work. Many refer to problem solving in non-mathematical contexts as *modelling*. It may be that situations/problems arise in another academic discipline, as one goes about one's everyday life, as a citizen in making sense of the world more widely, or in paid or unpaid employment. Indeed, it is recognised that in preparing for study in a discipline such as science or in preparing for employment particular content domains and ways of working mathematically are appropriate.

The Cambridge Framework allows for identification of context in the following context types:

- pure (the problem situation is in the world of mathematics)
- academic (the problems arise in the context of academic disciplines other than mathematics)
- everyday authentic (the problem situation might be met by someone in their everyday life using mathematics in ways that would commonly be used, for example in problems relating to personal finance)
- everyday artificial (problems posed in an everyday context using mathematics in ways that would not be typical in everyday practice)
- critical citizenship (for example, engaging mathematically with data communicated in the media)
- vocational (situations and problems relating to contexts involving employment).[18]

We are keen to seek opinion on whether these distinctions would be helpful, in particular in designing and distinguishing between assessments.

Technology

There are many different definitions of educational technology, and many different uses. Typically descriptions have been focused on the social and practical affordances such as the use of interactive white boards to promote classroom discussion, the use of graphical calculators as a mediating tool between groups of learners or the practical uses of portable equipment in a range of lesson types. We are less interested in this kind of use and more interested in the way in which software can impact on mathematical thinking. It is now possible for example, to interrogate large data sets and to model in a more sophisticated way than ever before. Just as calculators can enable young children to explore number patterns, so computer algebra systems (CAS) allow exploration of structures. Research about the use of CAS[19] shows that they provide material for conjecture and exploration. The power of such programmes is that many cases can be presented that are related in such a way that patterns can be generated and concepts deduced. For example, '*is $x^n - y^n$ divisible by $(x-y)$ for all n?*' This question is within the capability of a youngish students with CAS but would be tedious and error prone to ask at the same age without it.

Watson writes "The point is that ICT use turns possible mathematics questions on their heads, so that concepts are revealed by good explorations using ICT long before the technical skills allow the same questions to be asked using paper and pencil methods. Material for generalisation can be generated by dynamic methods such as spreadsheets – the generalisations are rapidly available instead of being merely stated and therefore have to be believed."[20] Assuming this is correct there are implications for the ordering of both the content and the first presentation of concepts in our Framework. We want to delve deeply into the research to determine whether we need to rethink our sequencing.

CAMBRIDGE MATHEMATICS FRAMEWORK

	NUMBER, OPERATIONS, ALGEBRA RELATIONS		
	FIRST EXPOSURE TO CONCEPTS	THRESHOLD TASKS	FIRST EXPOSURE TO CONCEPTS
STAGE 2B	Proof	Use algebra to support and construct arguments and proofs	
	Set, union, intersection, empty set	*Describe relationships between sets. (Data and uncertainty)*	
	Sequences as functions, common difference, ratio, nth term	Calculate arithmetic and geometric series, know common sequences	*Identity, inverse*
	Matrix arithmetic	Calculate with matrices, use to solve simultaneous equations	
		Solve quadratic equations, solve simultaneous linear and quadratic, solve inequaliaties algebraically. (Geometry)	
		Know, use, rearrange standard formulae (Geometry, measures)	
		Generalize and use rules of arithmetic to simplify, expand, factorise algebraic expressions	
	Direct inverse proportion	*Calculate with proportion (Measures)*	
		Convert, calculate with standard form (Measures)	
	Indices Surds	Calculate with indices and surds	
	Real, irrational, rational, numbers	Know common irrational numbers	
		Use and interpret upper and lower bounds (Measure)	
STAGE 2A		Set up and solve linear and two simultaneous linear equations	
		Simplify, collect like terms, expand and factorise simple expressions	
	Prime numbers	Find HCF, LCM	
	Significant figures	*Round numbers and calculations (Measure)*	Reflection rotation, translation enlargement, invariance
	Equivalence	Convert, compare fractions decimals, ratio, percentages	*Gradient intercept*
	Rate	*Calculate with rates (Measures)*	*Scale factors*
	Unequal division. Ratio notation	*Calculate with ratio (Measures)*	
	Percentage	Calculate with percentages	
	Reciprocal	Multiply, divide fractions	
		Multiply, divide decimals	
STAGE 1B			
	Sample space, variable	*Enumerate possibilities of combinations of two variables, (Data)*	
		Find pairs of numbers that satisfy equations with two unknowns	Area Perimeter Volume
	Unknown	Express missing number problems algebraically, use simple formulae	
		Generate and describe linear number sequences	
	Estimation	*Use approximationto check answers (Measures)*	
	Order of operations	Calculate multistep problems using four operations with whole numbers	
		Add subtract decimals, multiple and divide decimal by whole number (Money, measures)	
	Equivalence	Know and use equivalence between common fractions, decimals and percentages	Angle sums, parallel, perpendicular lines
	Equivalent fractions	Compare fractions, find fractions of whole and quantity, add subtract fractions	Scale factor
	Multiple, factor, prime	Count, multiply, divide multiples of whole numbers, find factors	Angle Symmetry Transformations
	Negative numbers	*Count, order compare integers, add subtract negative numbers (Measure)*	
STAGE 1A	Odd, even	Count in number sequences	Straight line, turn, relative positions
	Multiplication, equal division X, <division symbol> notation. Many to one correspondence	Multiply, divide multiples of 2, 5,10	*Exchange*
	Half, quarter, equal sharing	*Count halves, quarters, unit fractions to 1/10. Find unit fraction of whole, quantity (Measure)*	*Passage of time*
	Addition, subtractioninverse. Commutativity. Associativity. Equivalence =,<,> +,-	Add/subtract whole numbers	Length, mass, capacity
	Zero. Counting. Place Value. One-to-one correspondence	Count, order, compare whole numbers	

Italics indicate connection across subdomain

	GEOMETRY AND MEASUREMENT	DATA AND UNCERTAINTY	
	THRESHOLD TASKS	FIRST EXPOSURE TO CONCEPTS	THRESHOLD TASKS
	Use matrices to describe transformations (Number)		
	Represent and calculate translations using vectors		
	Interpret and use bearings		
	Sketch and interpret distance –time and velocity time graphs		
	Sketch and interpret quadratic functions, solve simultaneous linear and quadratic graphically, solve inequalities graphically (Algebraic relations)		
	Use and prove common angle properties of circles		
	Use trigonometric ratios and sine and cosine rules to calculate lengths and angles in triangles.		Infer distributions or populations from samples
	Sketch and use graphs of trigonometric functions	Cumulative frequency	Construct and interpret diagrams for grouped discrete and continuous data
	Use scaling to calculate lengths areas volumes of similar figures		Calculate and interpret conditional probabilities using expected frequencies
	Find surface areas, volumes for composite 3-d shapes	Exhaustive set sum to one	Calculate probability of independent and dependent combined events
	Recognise describe and perform transformations in the plane		
	Plot and interpret straight line graphs, interpret gradient as ratio or rate (Number, algebraic relations)		
	Recognise and use congruence and similarity (Number)	Scatter graphs correlation best fit	Interpret relationships of bivariate data
	Know prove and use Pythagoras' theorem	Measures of central tendency, range	Describe, interpret, illustrate and compare distributions of discrete continuous and grouped data
	Construct nets, elevations of 3-d shapes		Calculate probabilities of single, successive, combined events
	Measure and draw accurately, use ruler and compass constructions		Generate sample spaces for single combined, equally likely and mutually exclusive events
	Find lengths areas volumes for common 2-d and 3-d shapes	Equally likely, probability scale	Distinguish between experimental and theoretical probability
	Interpret simple maps (Number)		
	Use relationship between radius and diameter of circles		
	Plot, read coordinates in all four quadrants		
	Calculate rectangular and triangular area, rectangular perimeter, cuboid volume		
		Fair	Use the language of chance to describe everyday situations
	Order and calculate with angle in simple figures, on straight lines	Average, central measures	Find and interpret mean, mode, median
	Combine, enlarge and split shapes		***Interpret and construct tables including timetables (Measures)***
	Classify, draw 2-d and 3-d shapes		Interpret and distinguish between the uses of *pie charts*, line graphs, histograms *(Geometry)*
	Move between 2-d and 3-d representations	Carroll diagram, Venn diagram	Sort organise display describe simple data using two criteria
	Describe movement and position		
	Sort, order add subtract money (Number)		
	Tell and calculate time (Number)		
	Measure length, mass, volume using non-standard and common standard measures		
	Sort identify describe 2-d and 3-d shapes	***Pictogram, bar chart Data cycle***	***Sort organise display describe simple data using one criteria (Number)***

USING THE FRAMEWORK TO SUPPORT ASSESSMENT

Individual tasks

The full Framework will be evidence-based and therefore, in a way, as neutral as possible. It will also be a mapping of anything that is contained within mathematics curricula, no matter where they originate. That means that the elements of *any* assessment task could be mapped to the Framework.

Here are some examples:

1

Leo bought a balloon at the circus.
He gave the clown six coins to pay for it.

What could Leo have paid for the balloon?

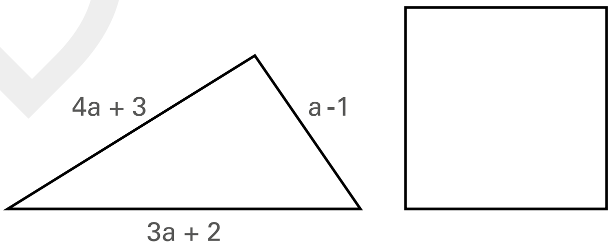
Which of your answers seems a reasonable amount to pay for a balloon?

Source: nrich.maths.org

Stage	1B
Content	Number, measures
Conceptual understanding	Relational
Working mathematically	Formulating, analysing, evaluating
Context	Authentic

2

The perimeter of the triangle is the same length as the perimeter of the square



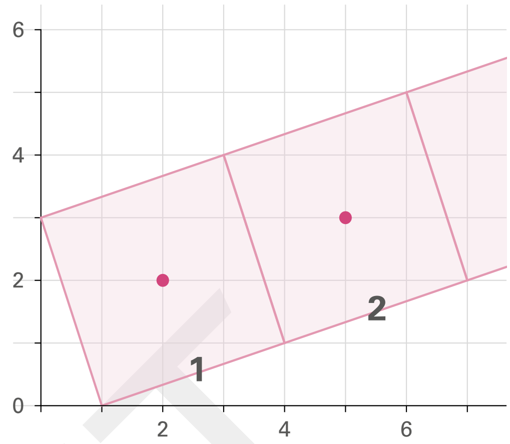
Find an expression for the length of one side of the square in terms of a.

Source: OCR

Stage	2A
Content	Number, procedures and algebraic relations
Conceptual understanding	Relational
Working mathematically	Formulating, analysing, interpreting
Context	Pure

3

Charlie has been drawing squares.



What will the coordinates of the centre of square number 3 be? How do you know?

Charlie wants to know where the centre of square number 20 will be.
Can you use the diagram above to help you to work this out?

Can you suggest a quick and efficient strategy for working out the coordinates of the centre of any square?

Would your strategy work if Charlie's sequence extended to the left?
….–2,–1,0,1,2,3….

Source: nrich.maths.org

Stage	2B
Content	Geometry, algebra relations
Conceptual understanding	Relational
Working mathematically	Analysing, evaluating
Context	Pure

4

What is the biggest triangle it is possible to draw inside a circle?

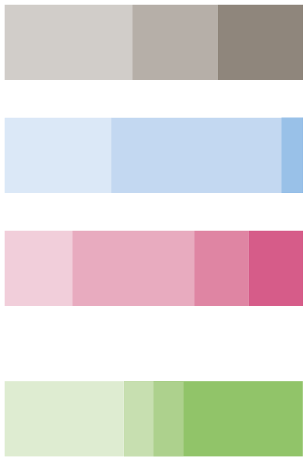
Source: Geoff Wake

Stage	2B
Content	Geometry, algebra relations
Conceptual understanding	Schematic
Working mathematically	Analysing, evaluating, communicating
Context	Pure

Groups of tasks

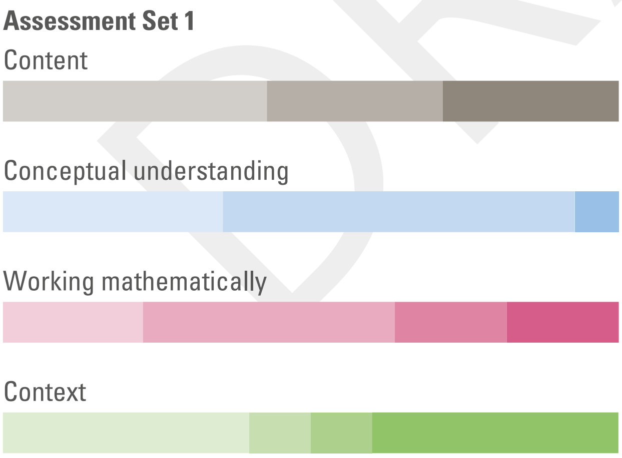
Collated information for a set of tasks can be drawn up in a table, and the spread within each dimension can be illustrated by a bar chart.

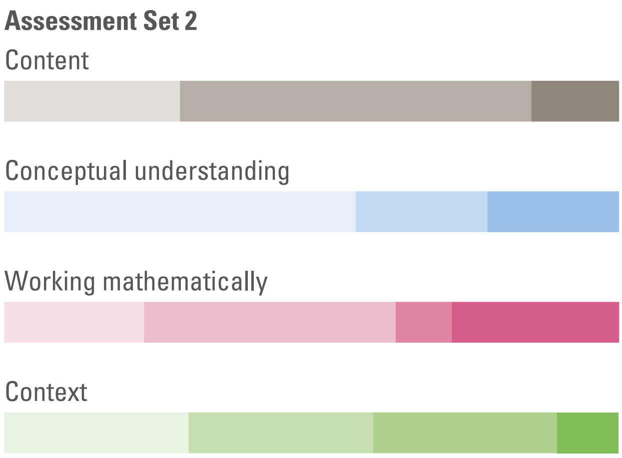
Task number		1	2	3	4	5	6	7	8	9	10
Domain	Number, procedures and algebraic relations	♦			♦	♦			♦	♦	♦
	Geometry and Measures		♦		♦		♦				♦
	Data and uncertainty			♦		♦		♦			♦
Conceptual understanding	Procedural	♦	♦		♦	♦		♦			
	Relational			♦	♦	♦	♦	♦	♦	♦	♦
	Schematic										♦
Working mathematically	Formulate	♦	♦			♦		♦			♦
	Analyse	♦		♦	♦	♦	♦	♦	♦	♦	♦
	Interpret and evaluate				♦			♦	♦		♦
	Communicate		♦						♦	♦	♦
Context	Pure	♦		♦		♦	♦				
	Academic		♦								
	Authentic				♦						
	Artificial							♦	♦	♦	♦
	Citizenship										
	Vocational										



This then gives a picture of the balance of the set of tasks, which means we can compare two sets to see if they are equivalent in nature – any assessments, good, bad or indifferent can be thus mapped to the Framework and compared.[21]

Comparing assessments





From these visuals we can see, for example, that Set 1 has a greater emphasis on number, relational understanding and interpretation than Set 2. Set 1 has more artificial contexts than Set 2.

Reverse engineering

This process is quite useful as a comparison and of course mimics in a rather crude way what Awarding Organisations already do. We could add in as many additional dimensions as necessary in order to make the process more refined.

But this process makes no value judgement on what an appropriate assessment for a *particular* purpose looks like. The balance across the dimensions will be different between, for example, an entry level qualification for a plumber and the entry level for access to further academic study or the assessment appropriate for an adult numeracy qualification. Cambridge Mathematics *will* have a view on what is the most appropriate profile for any particular assessment. These value judgements will be arrived at in partnership with the appropriate colleagues with the relevant expertise, both within and outside the University.

In summary, we think the Cambridge Mathematics Framework can support assessment design through:

- mapping individual assessment tasks to determine their characteristics
- mapping existing sets of assessments in order to give a picture of the within- and across-dimension balance in order to compare them against each other
- reverse engineering assessments to fit a particular profile

USING THE CAMBRIDGE MATHEMATICS FRAMEWORK TO MAP CURRICULA, RESOURCES AND PROFESSIONAL DEVELOPMENT

Using the Framework for assessments will support our understanding of how it can also be used for comparison and reverse engineering of curricula, resources and professional development. There will be many different possible curriculum pathways, each of which will flesh out a more detailed hierarchy of content. It is likely that there will be additional/replacement dimensions that will need to be drawn on, and similarly for resources and professional development. We have not yet explored the evidence and research base on this but look forward to building on the discussions from the launch to inform our direction of travel.

NOTES

1 Cambridge University Press, University of Cambridge Faculty of Education, University of Cambridge Faculty of Mathematics and Cambridge Assessment

2 We are indebted to Geoff Wake for providing much food for thought in the initial stages of the draft

3 PISA 2012 *Results: Excellence through equity (Volume II) giving every student the chance to succeed*, OCED Publishing

4 UK colleagues may wonder about the absence of Mechanics and Decision mathematics at the higher end of the content spectrum – as these are not common to all curriculums we will leave these until later in our deliberations, but they will be included.

5 *PISA 2015 Draft Mathematics Framework*, OCED

6 *Common European Framework for Languages: Learning, Teaching, Assessment Council of Europe*, European Council

7 Wikipedia http://en.wikipedia.org/wiki/Educational_stage (accessed January 2015)

8 Niss (1993) *Investigations into Assessment in Mathematics Education, New ICMI Study Series Volume II*

9 Ed Tech Now (2014) *Tim Oates: assessing without levels* http://edtechnow.net/2014/11/12/oates/#more-1146 (accessed January 2015)

10 Watanabe (2006) *The teaching and learning of fractions: a Japanese perspective in teaching children mathematics*, NCTM

11 See, for example, Meyer and Land (2003) *Threshold concepts and troublesome knowledge*, ETL Project, ESRC

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Cambridge Mathematics
1 Hills Road
Cambridge
CB1 2EU

Website: www.cambridgemaths.org
Email: info@cambridgemaths.org
Twitter: @CambridgeMaths #CamMaths